Metaphysics of Spacetime

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Today's plan

About the presentation

I will briefly present some results of my past and ongoing projects, and mention some future ones.

Overview

- Infinitesimal gunky space
- Smooth infinitesimals in physical geometry
- An algebraic approach to physical fields
- Ongoing & Future Work

Infinitesimal Gunky space[1]

Gunky space

Space is indefinitely divisible, and there are no indivisible regions.

Motivations:

- Point-sized regions do not play much role in physics.
- Point-sized regions cause difficulties in physical and mathematical formalism.

[1] "Infinitesimal Gunk," *Journal of Philosophical Logic* (2020).

Infinitesimal Gunky space

Standard formalism

Every region of 1d gunky space is represented by a "regular closed set" in the real line. (Intuitively: a set "fat" enough to have interiors.)

Problem

According to standard measure theory, a set with no interior can have a non-zero measure.

Infinitesimal Gunky space

My proposal

Every region is represented by a "regular closed set" in the hyperreal line featured by nonstandard analysis.

Technical Highlights:

- I defined a measure on space that satisfies "hyperfinite additivity." (hyperfinite is an infinite cardinal)
- I proved the topology is different from Roeper's proposal intended for all gunky space.

Infinitesimal Gunky space

Advantages:

- Every region has interiors. Every set with non zero measure, e.g., the fat Cantor set defined on the hyperreal line, has interiors, and is regular closed.
- It has a rich measure theory.

Smooth infinitesimals in physical geometry

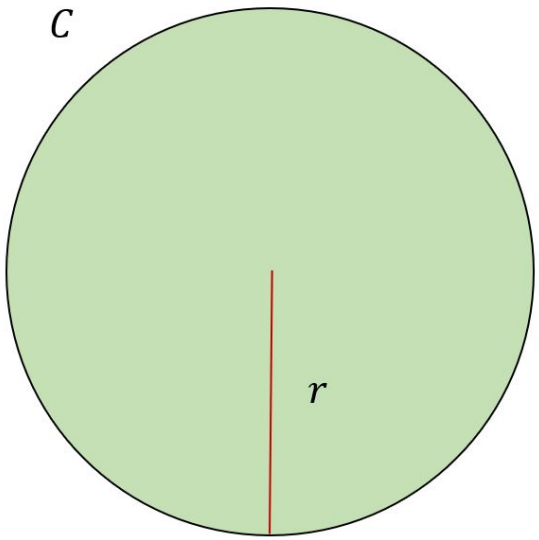
I explore a new theory of space with infinitesimal regions based on *smooth infinitesimal analysis*, which:[2]

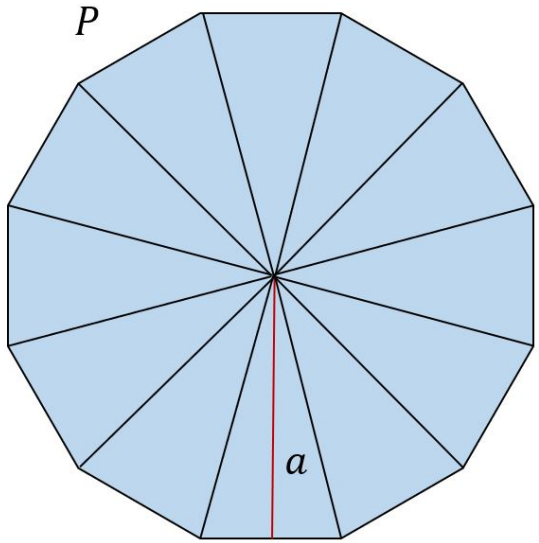
- regiments scientific reasoning involving infinitesimals.
- gives a new understanding of vectorial quantities.

[2] "Smooth Infinitesimals in the Metaphysical Foundation of Spacetime Theories," under review.

Infinitesimals in Geometry

A circle is a regular polygon with infinitesimal sides.



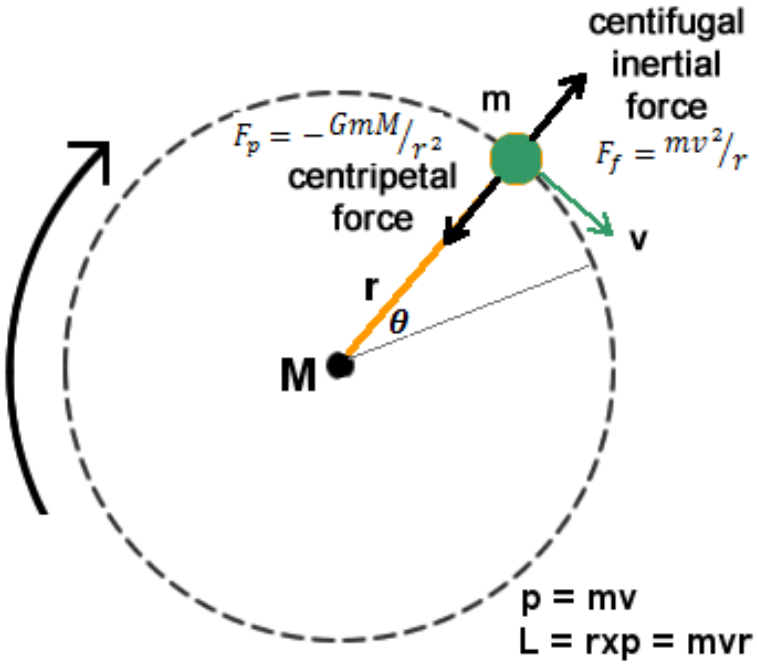


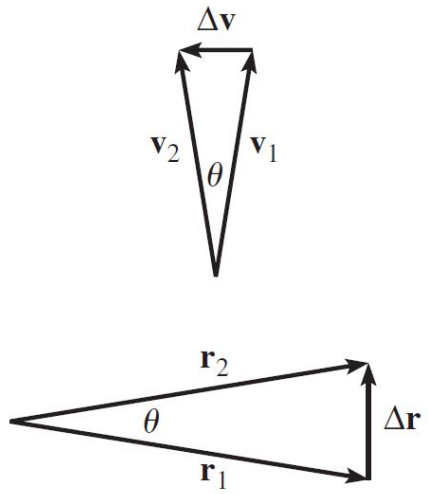
Area$= \frac{1}{2}rC$.

Area$= \frac{1}{2}aP$.

Infinitesimals in Physics

An example from classical mechanics:





$|\Delta \mathrm{v}|/v = |\Delta \mathrm{r}|/r$ (triangle similarity)

$a = v^2/r$

Smooth Infinitesimal Analysis

Smooth infinitesimal analysis (SIA) is a theory of infinitesimals that aims to regiment those ideas.

Problem: The theory uses intuitionistic logic, and is classically inconsistent.

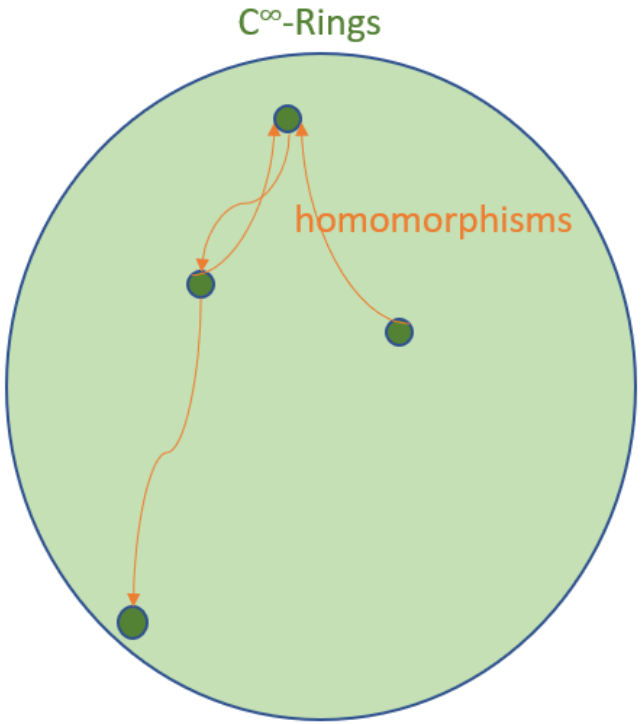
Example

SIA says: the set of nilpotent numbers is not $\{0\}$, but there are no non-zero nilpotent numbers.

My solution

Moerdijk and Reyes (1991) proposed categorical models for SIA under *sheaf semantics*.

The models consist of C^∞-*ring*s (roughly, an algebraic structure that have infinitely many operators).



My solution

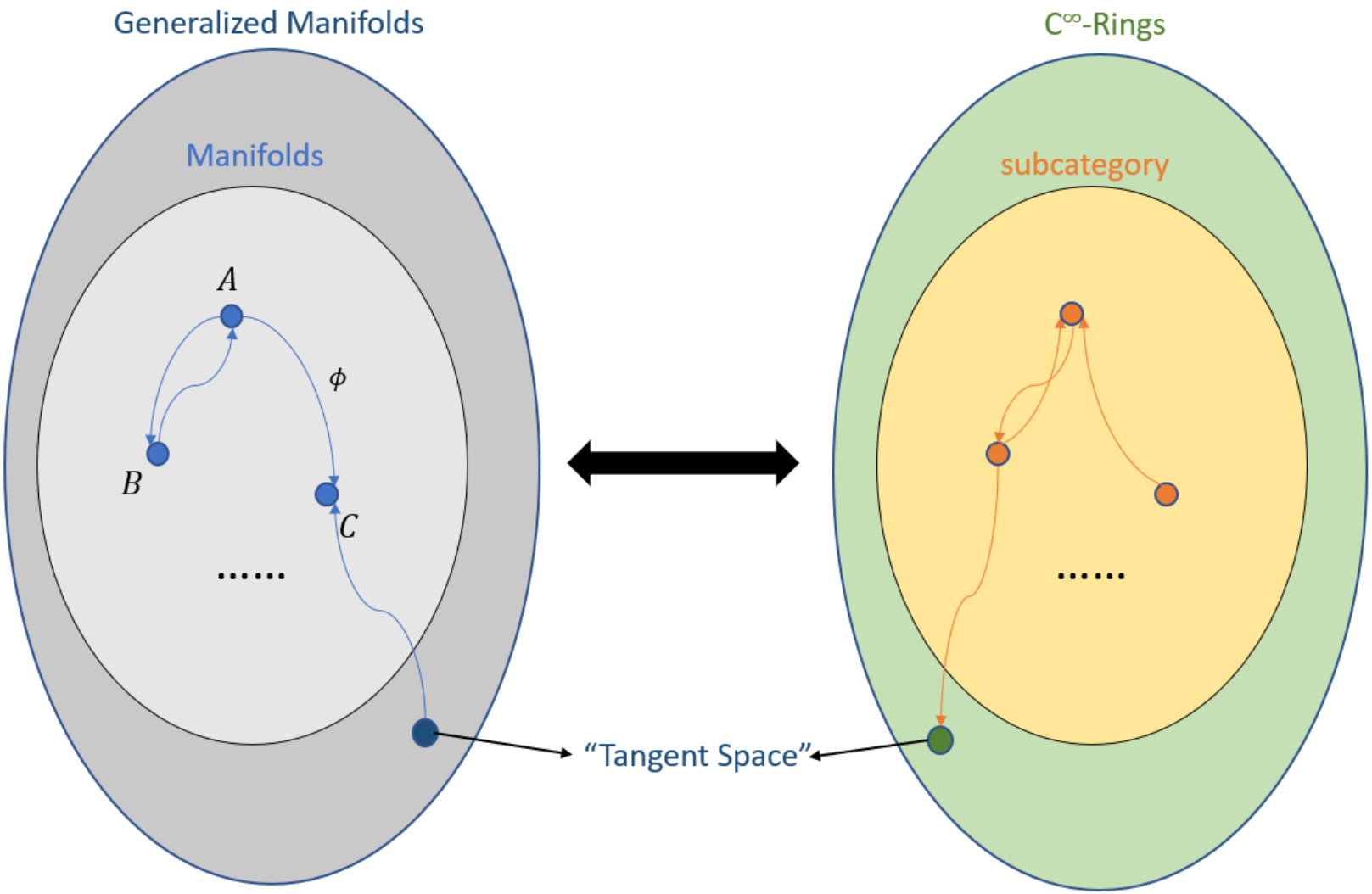
The semantic approach to SIA: one of those models represent space.

- A C^∞-ring represents a region of space.
- A homomorphism represents a parthood relation.

...

As a result, I obtain a classical theory of space with infinitesimal regions from SIA (with a mereology and differential structure).

Generalizing manifolds



An Algebraic Approach to Physical Fields[3]

Manifold Substantivalism

Physical fields are standardly defined on manifolds. This leads to the view that manifolds are real and represent spacetime.

Problems:

- Empirical underdetermination & radical indeterminism
- Quantum physics

[3] "Beyond Spacetime: an Algebraic Approach to all Physical Fields," (coauthored with Fritz) under review.

An Algebraic Approach to Physical Fields

Algebraicism

Physical fields exist at the fundamental level without an underlying manifold. The configurations of a field constitute an algebraic structure, e.g., an C^∞-*ring*.

But there is still a "ghost" manifold in this algebraicism. The basic algebraic entity is not a physical field recognized by current physics.

Our Solution

Field Algebra

The basic algebraic structure consists of all and only physical fields recognized by current physics (with the possibility to expand).

How do we build an algebra consisting of all types of physical fields? e.g. electromagnetic field is a one-form; the metric field is a tensor field; fermion fields are spinor fields.

Tools from category theory:

- Fields $\rightarrow$ Functors
- Interactions between fields $\rightarrow$ Natural transformations
- Fields $\rightarrow$ the field algebra defined by "natural operators"

Other Projects

A Defense of Dynamicism

I defend the view that geometric features of physical reality are not fundamental and can be derived from dynamic laws. (Under preparation; draft available.)

Where Does Effective Spacetime Fall short

I highlight the appeals of the EFT approach to physics, but argue that the approach to general relativity fails as a realistic theory. (Under preparation; draft available.)

Future Projects

Nominal Individuals in Ontic Structuralism

I aim to propose a formulation of ontic structuralism, the view that structures are more fundamental than objects, using nominal sets.

Regarding "The Weyl Tile argument"

I aim to point out that the Weyl tile argument against discrete spacetime is based on a faulty assumption about how distances at the large scale emerge from the microscopic scale.